Quiz #6, Math 10A, Section ____

Name: _____

1) a) Calculate $\int \ln(u)\,du$ (2 points)

b) Calculate $\int_{e^2}^{e^3} \frac{\ln(\ln(x))}{x}\,dx$ (2 points)

2) Calculate $\int x^2 \cdot e^{-x}\,dx$ (2 points)

3) Approximated $\int_{-2}^{2} |x|\,dx$ using Riemann sums for $n=4$, indicate if you are using midpoints, left or right endpoints (2 points)

1) a) By integration by parts

$$\int \ln(u)\,du = u\cdot\ln(u) - \int du = u\cdot\ln(u) - u + C$$

$U = \ln(u)$ | $dU = \frac{du}{u}$

$dv = du$ | $V = u$

b) if $u = \ln(x)$, $\frac{dx}{x} = du$ (u-substitution)

$$\int_{e^2}^{e^3} \frac{\ln(\ln(x))}{x}\,dx = \int_2^3 \ln(u)\,du = \left(u\cdot\ln(u) - u\right)\Big|_2^3$$

$$= 3\ln(3) - 3 - 2\ln(2) + 2 = 3\ln(3) - 2\ln(2) - 1$$

2) By integration by parts

$$\int x^2 e^{-x}\,dx = -x^2e^{-x} - \int -2xe^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}\,dx \quad \ldots (1)$$

$U = x^2$ | $dU = 2x\,dx$

$dV = e^{-x}\,dx$ | $V = -e^{-x}$

$$\int 2xe^{-x}\,dx = -2xe^{-x} - \int -e^{-x}\,dx = -2xe^{-x} + 2\int e^{-x}\,dx$$

$U = 2x$ | $dU = 2dx$

$dv = e^{-x}$ | $V = -e^{-x}$

$$= -2xe^{-x} - 2e^{-x} + C \quad \ldots (2)$$

(2) in (1) $\int x^2e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

3) Left endpoint $f(x) = |x|$

$$f(-2)\cdot\left(\frac{4}{4}\right) + f(-1)\left(\frac{4}{4}\right) + f(0)\cdot\left(\frac{4}{4}\right) + f(1)\cdot\left(\frac{4}{4}\right)$$

$$= 2 + 1 + 0 + 1 = 4$$

Right endpoint

$$f(-1)\left(\frac{4}{4}\right) + f(0)\left(\frac{4}{4}\right) + f(1)\left(\frac{4}{4}\right) + f(2)\left(\frac{4}{4}\right)$$

$$= 1 + 0 + 1 + 2 = 4$$

Midpoint:

$$f(-1,5)\left(\frac{4}{4}\right) + f(-0,5)\left(\frac{4}{4}\right) + f(0,5)\cdot\frac{4}{4} + f(1,5)\left(\frac{4}{4}\right)$$

$$= 1,5 + 0,5 + 0,5 + 1,5 = 4$$